



## Midbroken Reinforced Concrete Shear Frames Due to Earthquakes

*a hysteretic model to quantify damage at the storey level*

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*Publication date:*  
1996

*Document Version*  
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*

Köylüoglu, H. U., Cakmak, A. S., & Nielsen, S. R. K. (1996). *Midbroken Reinforced Concrete Shear Frames Due to Earthquakes: a hysteretic model to quantify damage at the storey level*. Dept. of Building Technology and Structural Engineering. Structural Reliability Theory Vol. R9630 No. 143

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# MIDBROKEN REINFORCED CONCRETE SHEAR FRAMES DUE TO EARTHQUAKES

A Hysteretic Model to Quantify Damage at the Storey Level

H. Uğur Köylüoğlu

*College of Arts and Sciences*

*Koç University, TR-80860 İstinye, İstanbul, Turkey*

Ahmet Ş. Çakmak

*Department of Civil Engineering and Operations Research*

*Princeton University, Princeton, NJ 08544, USA*

Søren R. K. Nielsen

*Department of Structural Engineering and Building Technology*

*Aalborg University, DK-9000 Aalborg, Denmark*

## ABSTRACT

A non-linear hysteretic model for the response and local damage analyses of reinforced concrete shear frames subject to earthquake excitation is proposed, and, the model is applied to analyse midbroken reinforced concrete (RC) structures due to earthquake loads. Each storey of the shear frame is represented by a Clough and Johnston hysteretic oscillator with degrading elastic fraction of the restoring force. The local damage is numerically quantified in the domain  $[0,1]$  using the maximum softening damage indicators which are defined in closed form based on the variation of the eigenfrequency of the local oscillators due to the local stiffness and strength deterioration. The proposed method of response and damage analyses is illustrated using a sample 5 storey shear frame with a weak third storey in stiffness and/or strength subject to sinusoidal and simulated earthquake excitations for which the horizontal component of the ground motion is modelled as a stationary Gaussian stochastic process with Kanai-Tajimi Spectrum, multiplied by an envelope function.

## 1. INTRODUCTION

The purpose of this study is to derive a non-linear hysteretic mechanical formulation to quantify local damage at the storey level in reinforced concrete (RC) multi-degree-of-freedom (MDOF) shear frames subject to earthquakes. The motivation stems from the collapses occurred at the intermediate storeys of some RC structures in the recent earthquakes of Kobe, Japan, January 1995 and Dinar, Turkey, October 1995. The photograph of a midbroken structure during the Dinar Earthquake of October 1995 is shown in Figure 1. Intermediate storey collapses are basically due to the buckling of columns caused by severe horizontal ground accelerations coupled with vertical accelerations. In this study, only the effects of horizontal ground accelerations are considered using a shear frame model. The physical local damage in RC structures subject to severe seis-



mic excitation is attributed to micro-cracking and crushing of concrete, yielding of the reinforcement bars and bond deterioration at the steel-concrete interfaces. To the extent that RC structures can be modelled by non-linear mechanical theories, local damage at a cross-section of the structure can adequately be measured by the degradation of bending stiffness and moment capacity of the cross-section. In a shear frame analysis, this is numerically tantamount to a reduction in the local stiffness and strength. In what follows, such decreases are used to define a measure to quantify local damage.

The mechanical theory is especially designed to explain the failures at the intermediate floors quantitatively based on a shear frame analysis, and may be used for the analysis of RC structures with weak intermediate floors so that precautions can be taken to strengthen the necessary weak parts of the structure. The weakness of the intermediate storey is explained by a small local stiffness per unit mass or less strength. These reductions affect the behaviour of the material under cyclic loading, and are considered to be linear and non-linear measures of local weakness, respectively. Local weakness in terms of stiffness defines the slope of the stress-strain curve, and can be attributed to sudden changes in column cross-section due to the nature of the design, extending extra storeys on top of the already designed buildings, using long slender columns at the intermediate storeys for architectural beauty, having weak column-beam connections at the column bases, modelling errors, heavy storeys or roof, etc. A weakness in terms of strength is tantamount to a lower yield level, and is due to usage of poor quality concrete and steel at the column, beam connections, sudden change of the amount and material of reinforcement, sudden change of materials, e.g. from concrete to steel, etc.

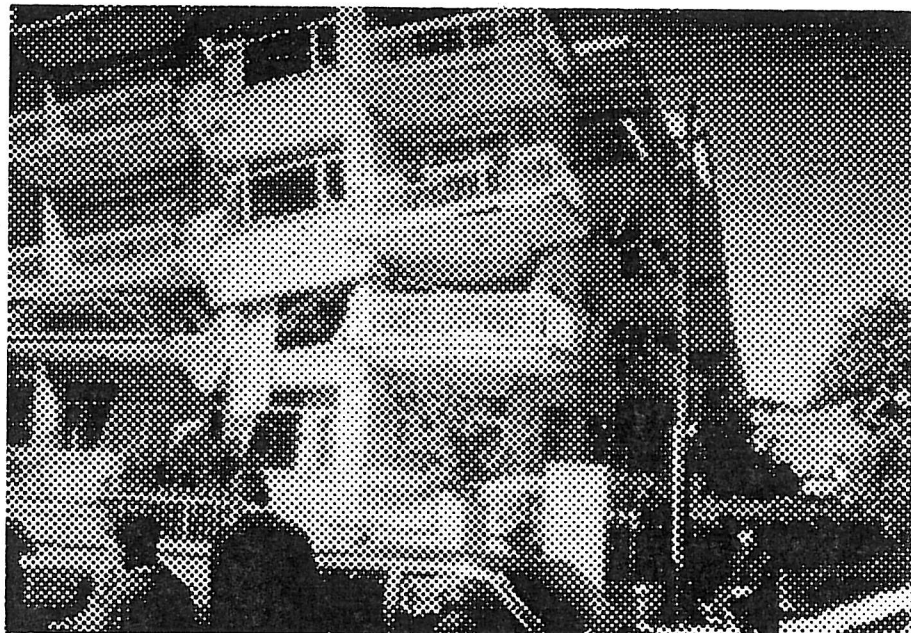


Figure 1. Collapse of an intermediate floor due to the October 1995 Earthquake, Dinar, Turkey. (Photograph taken by Prof. Hasan Boduroğlu of İstanbul Technical University, Turkey).



Local damage indicators are response quantities characterizing the damage state of a part of the structure after an earthquake excitation, and such can be used for structural analysis and in decision-making during the design phase, or in case of post-earthquake reliability and repair problems. In serving these purposes, the damage indicators should, at least, be observable for practical purposes and computable for quantitative analysis work, be a non-decreasing function of time unless the structure is repaired or strengthened, possess a failure surface (serviceability or ultimate limit state) to separate safe states from the unsafe ones and carry Markov property so that post-earthquake reliability estimates for a partly damaged element or structure can be estimated solely from the latest recorded value of the damage indicator.

The maximum softening damage indicators (MSDI) measure the maximum relative reduction of the vibrational frequencies for an equivalent linear system with slowly varying stiffness properties during a seismic event, hence, display the combined damaging effects of the maximum displacement ductility of the structure during extreme plastic deformations and the stiffness deterioration in the elastic regime, the latter effect being referred to as final softening. The introduction of the one-dimensional modal MSDI based on an equivalent linear single-degree-of-freedom (SDOF) system fit to the first mode of the RC building as a global damage indicator is due to DiPasquale and Çakmak (1990). The excitation and displacement response time series of a single position on the building are the only required observations for the one-dimensional modal MSDI. The applicability of the index was analysed based on data from shake table experiments with RC frames performed by Sözen and his associates (Çeçen 1978, Healey and Sözen 1979). The modal MSDI concept has also been generalized to MDOF models along with the associated damage localization problem, Nielsen et al. (1993). The Markov property of the modal MSDI chains for SDOF and 2DOF models was tested numerically by means of Monte Carlo simulations, Nielsen et al. (1992, 1993), and it was concluded that the modal MSDI fulfils Markov property with sufficient accuracy.

For prediction purposes, Köylüoğlu et al. (1994) proposed a hysteretic model suited to the first mode such that the first mode is modelled as a Clough and Johnston hysteretic oscillator, (Clough and Johnston, 1966), with degrading elastic fraction of the restoring force, subject to seismic excitation. This hysteretic model is generalized for MDOF systems to calculate local, modal and overall damage by representing each storey of the shear frame as a Clough and Johnston hysteretic oscillator recently, Köylüoğlu et al. (1995). The local MSDI is defined in a closed form based on the variation of the eigenfrequency of the local oscillators due to the local stiffness and strength deterioration. The MDOF hysteretic model for RC shear frames is derived, and numerical analysis is carried out for a sample shear frame. The shear frame is subject to sinusoidal input matching the first 3 frequencies of the shear frame and simulations of the earthquake excitation. Finally, the relationship between local, modal and overall damage indices is investigated statistically.

Damage indicators based on the changes in the natural frequencies of structures have also been used by other researchers, e.g. Hassiotis and Jeong (1993), Jajela and Soeiro (1990). Both studied the determination of the local damage based on information about the changes in the eigenfrequencies of the structure. Hassiotis and Jeong (1993) pro-



posed a quadratic optimization formulation to solve the arising undetermined equations with a suitable optimization criterion. They concluded that reduction in stiffness up to 40 per cent at single and multiple sites can be detected when perfectly measured data is used. It is also noted that when the data used is polluted randomly, light damage cannot be detected.

Rahman and Grigoriu (1994) related the hysteretic behaviour of the storey columns to the storey level degree of freedom analytically. They derived such relationships using hysteretic constitutive laws and quantified damage using an energy-based rule. Based on numerical studies, it is concluded that the global model, the storey level one, provides good estimates of seismic response quantities compared to the local model, the storey columns.

In this work, the hysteretic model for MDOF shear frames is defined, and the response and damage analyses are performed for three 5 DOF shear frame. All properties of the shear frames are taken the same except that the second is weak in stiffness and the third is weak in strength at the third storey level. Sensitivity studies are performed to relate such weaknesses to the eigenfrequencies, response and local damage characteristics.

## 2. HYSTERETIC MODEL FOR MDOF SHEAR FRAMES

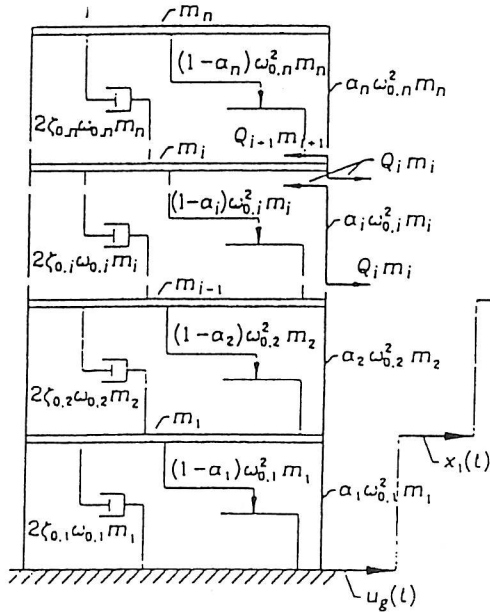


Figure 2. MDOF shear frame with local hysteretic oscillators.

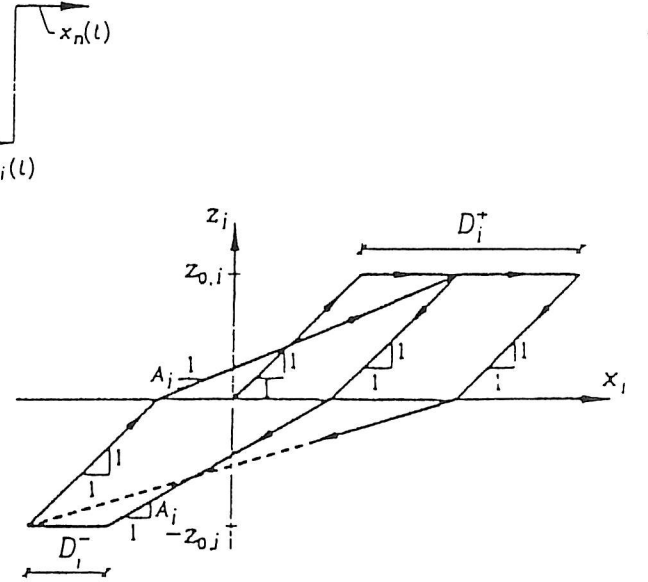


Figure 3. Clough-Johnston hysteretic model.

Consider an  $n$  storey RC shear frame. The relative displacement between the  $i$ th and  $(i + 1)$ th storeys is designated as  $x_i$ , and  $x_1$  signifies the displacement of the first storey relative to ground surface excited by the horizontal ground acceleration  $\ddot{u}_g$ , see Figure 2. For simplicity, the time dependence of  $x, \ddot{u}_g$  etc. is not explicitly shown in the following notation. With reference to the shear model shown in Figure 2, the relative displacement  $x_i$  between the  $i$ th and  $(i + 1)$ th storey is assumed to cause a shear force

of magnitude  $Q_i m_i$  where  $m_i$  is the storey mass. The equations of motion in terms of the relative displacements can be written as

$$\left. \begin{aligned} \ddot{x}_1 &= \mu_2 Q_2 - Q_1 - \ddot{u}_g & , \quad t > 0 \\ \ddot{x}_i &= \mu_{i+1} Q_{i+1} - (\mu_i + 1) Q_i + Q_{i-1} & , \quad t > 0 \quad , \quad i = 2, 3, \dots, n-1 \\ \ddot{x}_n &= -(\mu_n + 1) Q_n + Q_{n-1} & , \quad t > 0 \\ x_i(0) &= \dot{x}_i(0) = 0 & , \quad i = 1, 2, \dots, n \end{aligned} \right\} \quad (1)$$

$$\mu_i = \frac{m_i}{m_{i-1}} \quad , \quad i = 2, 3, \dots, n \quad (2)$$

$$Q_i = 2\zeta_{0,i}\omega_{0,i}\dot{x}_i + \omega_{0,i}^2(\alpha_i x_i + (1 - \alpha_i)z_i) \quad , \quad i = 1, 2, \dots, n \quad (3)$$

$$\dot{z}_i = k(\dot{x}_i, z_i, D_i; z_{0,i})\dot{x}_i \quad , \quad t > 0 \quad , \quad z_i(0) = 0 \quad , \quad i = 1, 2, \dots, n \quad (4)$$

$$\dot{D}_i = g(\dot{x}_i, z_i; z_{0,i})\dot{x}_i \quad , \quad t > 0 \quad , \quad D_i(0) = D_{0,i} \quad , \quad i = 1, 2, \dots, n \quad (5)$$

$$\alpha_i = \left( \frac{2z_{0,i}}{2z_{0,i} + D_i} \right)^{n_{0,i}} \quad , \quad i = 1, 2, \dots, n \quad (6)$$

$$\begin{aligned} k(\dot{x}_i, z_i, D_i; z_{0,i}) &= H(z_i) \{ A_i H(\dot{x}_i) (1 - H(z_i - z_{0,i})) + H(-\dot{x}_i) \} + \\ &H(-z_i) \{ A_i H(-\dot{x}_i) (1 - H(-z_i - z_{0,i})) + H(\dot{x}_i) \} \quad , \quad i = 1, 2, \dots, n \end{aligned} \quad (7)$$

$$g(\dot{x}_i, z_i; z_{0,i}) = H(\dot{x}_i)H(z_i - z_{0,i}) - H(-\dot{x}_i)H(-z_i - z_{0,i}) \quad , \quad i = 1, 2, \dots, n \quad (8)$$

$$A_i = \frac{z_{0,i}}{z_{0,i} + D_i} \quad , \quad i = 1, 2, \dots, n \quad (9)$$

$$H(x) = \begin{cases} 1 & , \quad x \geq 0 \\ 0 & , \quad x < 0 \end{cases} \quad (10)$$

Equation (1) is derived using Newton's second law of motion applied for each storey mass and next solved for the interstorey displacements. Equation (2) defines  $\mu_i$  which is the ratio of the lumped mass at the two consecutive storeys.  $2\zeta_{0,i}\omega_{0,i}m_i$  and  $\omega_{0,i}^2m_i$



appearing in equation (3) are, respectively, the linear viscous damping and initial elastic spring stiffnesses between the storeys.  $\alpha_i(D_i)$  is the elastic fraction of the restoring force which is a function of damage.  $z_i$  is the hysteretic component which is modelled using the Clough-Johnston hysteretic model.  $z_i = +z_{0,i}$ ,  $z_i = -z_{0,i}$  signify the yield levels.  $k(\dot{x}_i, z_i, D_i; z_{0,i})$  is a non-analytic function describing the state dependent stiffness of the hysteretic model on the component  $z_i$ . The stiffness degrading hysteretic constitutive law of the model can be represented as shown in Figure 3. The Clough-Johnston model deals with the stiffness degradation by changing the slope  $A_i$  of the elastic branches as the accumulated plastic deformations  $D_i^+$  and  $D_i^-$  at positive and negative yielding increase as shown in Figure 3.  $D_i = D_i^+ + D_i^-$  is the total accumulated plastic deformation. For loading branches, the slope  $A_i$  is selected such that the elastic branch always aims at the previous unloading point with the other sign. At unloadings, the slope is 1.  $D_{0,i}$  is the initial value of the total accumulated damage which is zero before the first earthquake hits and is assumed to be determined from previous earthquake and displacement response records for the succeeding earthquakes.  $H(x)$  is the unit step function.

A novelty of the present model primarily stems from the modelling of  $\alpha_i(D_i)$  as a non-increasing function of the damage parameter  $D_i$ . Since,  $\alpha_i(D_i)$  measures the fraction of the restoring force from linear elastic behaviour, this fraction must decrease as larger and larger parts of the structure become plastic. Note that for an undamaged structure,  $\alpha_i(D_i(0)) = 1$ , and, unless there is damage, still  $\alpha_i(0) = 1$ . The dependence of  $\alpha_i(D_i)$  on  $D_i$  as indicated by equation (6) has been selected to fulfil this boundary condition. The relative success of the model (1)-(10) in reproducing actually recorded displacement time series in the earlier work of the authors is primarily due to this modelling (Köylüoğlu et al. 1994).

The hysteretic parameters  $z_{0,i}$ ,  $n_{0,i}$  are assumed to be known from structural tests of the columns. The  $3n - 1$  linear parameters  $\omega_{0,i}$ ,  $\zeta_{0,i}$ ,  $\mu_i$  can be obtained from a structural analysis or via non-destructive testing.

It should be noted that the Clough-Johnston (1966) hysteretic model was originally designed for RC beam-columns. The differential description of the model was first introduced by Minai and Suzuki (1985) for SDOF systems.

### 3. LOCAL MAXIMUM SOFTENING DAMAGE INDICATORS

In the hysteretic model for the  $i$ th relative displacement, the instantaneous slope of the hysteretic curve defines the variations of the instantaneous stiffness. For the Clough-Johnston model, the instantaneous slope is  $A_i(t)$  for loading branches, 1 for unloading branches and 0 when yielding occurs. Therefore, instead of instantaneous softening, an average softening value is defined using the average slope  $\bar{m}_i$  in a hysteresis loop, the slope of the line through extreme points,

$$\bar{m}_i = \frac{2z_{0,i}}{2z_{0,i} + D_i(t)} \quad (11)$$

The loop-averaged softening  $S_i(t)$  is

$$S_i(t) = 1 - \sqrt{\frac{2z_{0,i}}{2z_{0,i} + D_i}(1 - \alpha_i) + \alpha_i} \quad , \quad i = 1, 2, \dots, n \quad (12)$$

$S_i(t)$  is a local damage indicator displaying the damaging effects of the local plastic deformations. As seen from (12),  $S_i(t)$  is non-decreasing during a seismic event and fully correlated to the total accumulated plastic deformation  $D_i(t)$ . It is also easily numerically computable.

The  $i$ th local MSDI  $S_{M,i}$  is defined as the maximum of  $S_i(t)$  which is the final value of  $S_i(t)$  during an earthquake.

$$S_{M,i} = \max S_i(t) \quad , \quad i = 1, 2, \dots, n \quad (13)$$

The local damage indicator,  $S_i(t)$ , takes on numerical values between 0 and 1, 0 denoting no local damage in the columns and 1 meaning total collapse of columns under the  $i$ th storey. Moreover, the local stiffness reduction percentage is functionally related to  $S_i(t)$  as  $(1 - S_i(t))^2$ , e.g.  $S_i(t) = 0.4$  means that the local stiffness has been reduced to 36 per cent of the initial local stiffness at the  $i$ th storey. Not only for  $S_{M,i} = 1$  but also for large values of  $S_{M,i}$ , one can assume the total collapse of columns, due to buckling.

#### 4. NUMERICAL INVESTIGATIONS

Five storey RC shear frames are used to demonstrate the abilities of the hysteretic model to detect and quantify local damage in shear frames subject to severe excitations. First, for comparison, three structures with minor differences, listed below, subject to four different excitations are analysed. Taking the first structure as a base for comparisons, the second structure has a weak third storey in terms of stiffness, and the third structure has a weak third storey in terms of strength. The excitations considered are sine excitations exciting the first and second modes of the structure and non-stationary pseudo-random excitations which are stationary time series with Kanai-Tajimi spectrum multiplied by an envelope function.



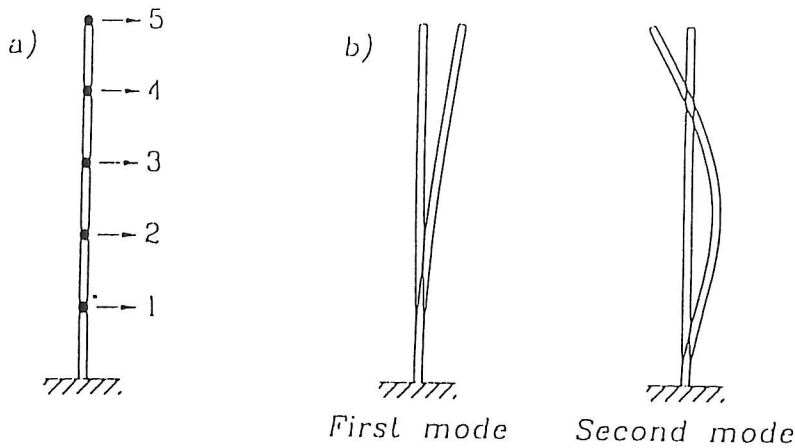


Figure 4. a) 5 DOF model for 5-storey RC shear frame. b) The first two mode shapes.

For the first structure, all storeys are assumed to have the same mass, stiffness and damping characteristics. Thus,  $\mu_i = 1$  and the frequencies, damping ratios of the local oscillators are taken as  $\omega_{0,i} = 7\pi \text{ sec}^{-1}$ ,  $\zeta_{0,i} = 0.03$  for  $i = 1, 2, \dots, n$ . The non-linear parameters are also assumed to be the same for all storeys and taken as  $z_{0,i} = 24 \text{ mm}$  and  $n_{0,i} = 0.8$  for  $i = 1, 2, \dots, n$ . The yield level 24 mm is found to be reasonable by the authors, since it corresponds to a horizontal strain of 0.008 for a storey of height 3 m. All properties of the second structure are the same as for the first, except that the circular frequency of the third storey is equal to 50 per cent of the circular frequency of the other storeys, e.g.  $\omega_{0,3} = (0.5) 7\pi \text{ sec}^{-1}$ . Physically, this can be due to about 30 per cent reduction in the dimensions of the column cross-section. All properties of the third structure are the same as for the first, except that the yield level of the third storey is equal to the 50 per cent of the yield level of the other storeys, e.g.  $z_{0,3} = (0.5) 24 \text{ mm}$ .

With these parameters, a linear eigenvalue analysis is performed, and the modal eigenfrequencies of the undamaged structures are determined. The eigenfrequencies and periods are listed in Table 1. Table 1 shows that considering linear properties, structures 1 and 3 are the same, and that the eigenfrequencies of the first and second structure are close, within 10 per cent difference, even though the second structure has a significantly weaker third storey in stiffness. This states that a local weakness in stiffness of an existing structure cannot be detected easily by non-destructive tests such as crude ambient vibrations test as the 10 per cent difference in eigenvalues can also be attributed to the experimental errors.

Table 1. Eigenfrequencies and periods of the 5-storey shear frame				
Mode Number	Structures 1 and 3		Structure 2	
	Frequency (Hertz)	Period (sec)	Frequency (Hertz)	Period (sec)
1	0.99620	1.00381	0.90404	1.10615
2	2.90791	0.34389	2.76285	0.36195
3	4.58403	0.21815	4.09768	0.24404

The structures are subject to 2 different sinusoidal loads as horizontal base accelerations with increasing amplitude and different frequencies. These excitations are chosen to excite the first and second modes of the undamaged structure, respectively. The sine excitations reach an amplitude of  $1g$  after 10 seconds and the system is led to eigenvibrations after that to observe the eigenvibrations with a large period due to damage and softening in the local oscillators. During such severe excitations, plastic deformations occur and the structure is damaged. The sine excitation  $\sin(6.28t)$ , and, the corresponding response and softening time series for three structures are given in Figures 5-10. Local MSDI are calculated for the two inputs and the results are tabulated in Table 2. At a local weakness in stiffness or strength, plastic deformations occur easily, the relative displacements are large and the excitation energy is absorbed. From Figures 5-10 and Table 2, as expected, the model captures all of these behaviours and local MSDI values quantify them. Moreover, in Figures 7 and 9, it is clear that during the eigenvibrations, the upper and lower parts of the structure behave independently due to significant plastic deformations at the third storey level.

Table 2. Local MSDI due to the sine excitations						
Excitation	Structure	$S_{M,1}$	$S_{M,2}$	$S_{M,3}$	$S_{M,4}$	$S_{M,5}$
$\sin(6.28t)$	Structure 1	0.506	0.465	0.287	0.097	0.000
	Structure 2	0.029	0.006	0.645	0.000	0.000
	Structure 3	0.188	0.140	0.713	0.023	0.000
$\sin(18.21t)$	Structure 1	0.136	0.000	0.000	0.114	0.010
	Structure 2	0.123	0.000	0.166	0.000	0.000
	Structure 3	0.146	0.000	0.165	0.085	0.005

Table 2 shows that the maximum damage takes place if the first mode is excited when the amplitudes of the 2 excitations are the same. This is the utmost dangerous excitation for the structure. Figures 5-10 and Table 2 are consistent with the mode shapes for the first structure. The first mode shape leads to the fact that the first storey relative displacement should be maximum and the maximum damage should take place in the lower part of the structure, especially in the first storey when the first mode is excited. The second mode shape describes the behaviour observed in Table 2 row 4 that the maximum damage should take place at the first and fourth storeys where the second eigenvector has large relative changes in the components. When there is a weakness in the structure in terms of stiffness or strength, MSDI values show that damage will be concentrated in the weak area, see the large values of  $S_{M,3}$  in Table 2 rows 2, 3, 5 and 6. According to Table 2, the excitation of the second mode, however, causes less damage in the structure even in the weak neighbourhoods.

Next, earthquake excitations are studied. The ground excitation,  $\ddot{u}_g(t)$  is modelled as a stationary stochastic Gaussian process  $V(t)$  with Kanai-Tajimi spectrum multiplied by an envelope function  $E(t)$ .



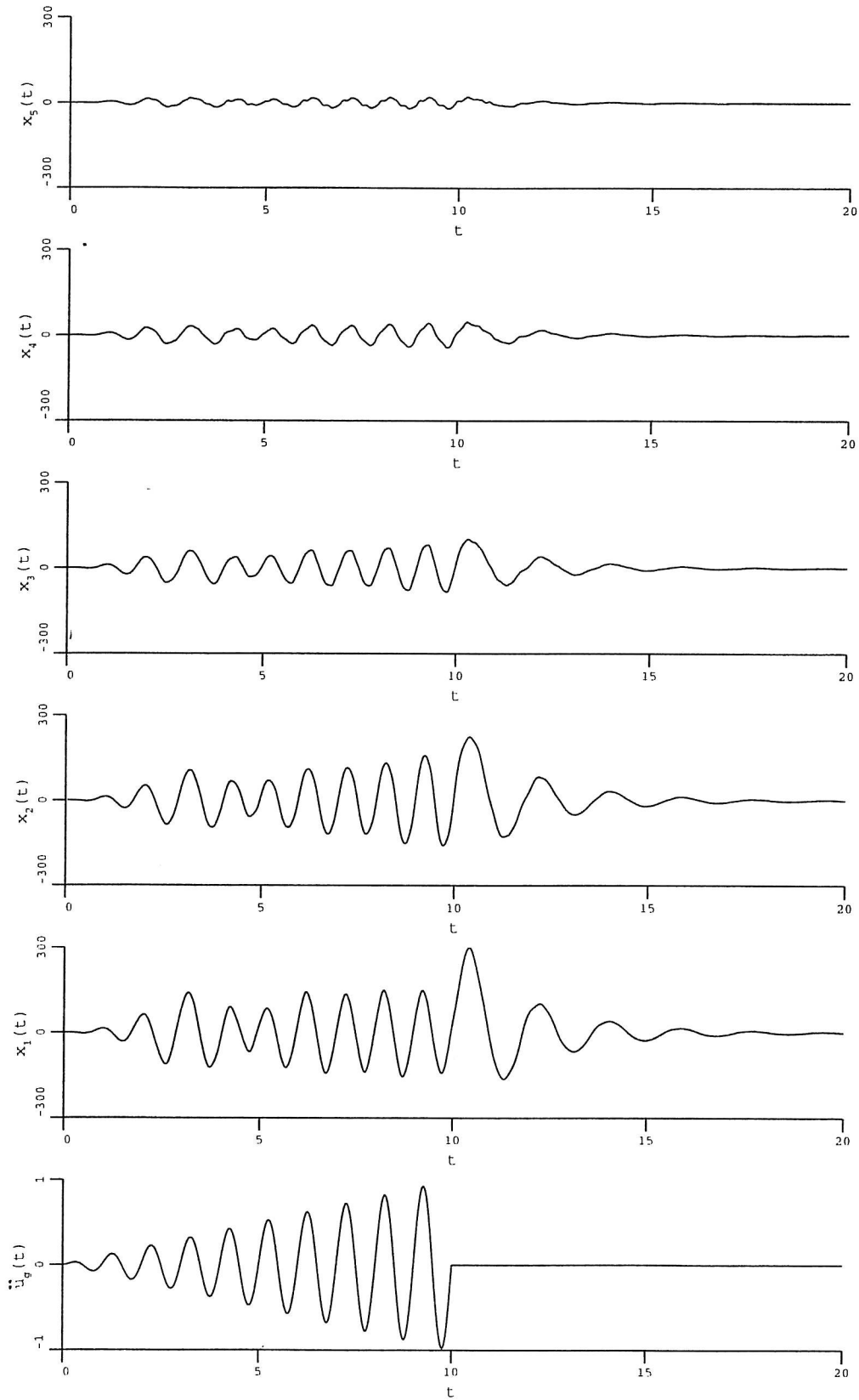


Figure 5. The sine ground acceleration (normalized to  $g$ ) exciting the first mode and the response of the first structure in terms of relative displacements  $x_i(t)$  in mm.

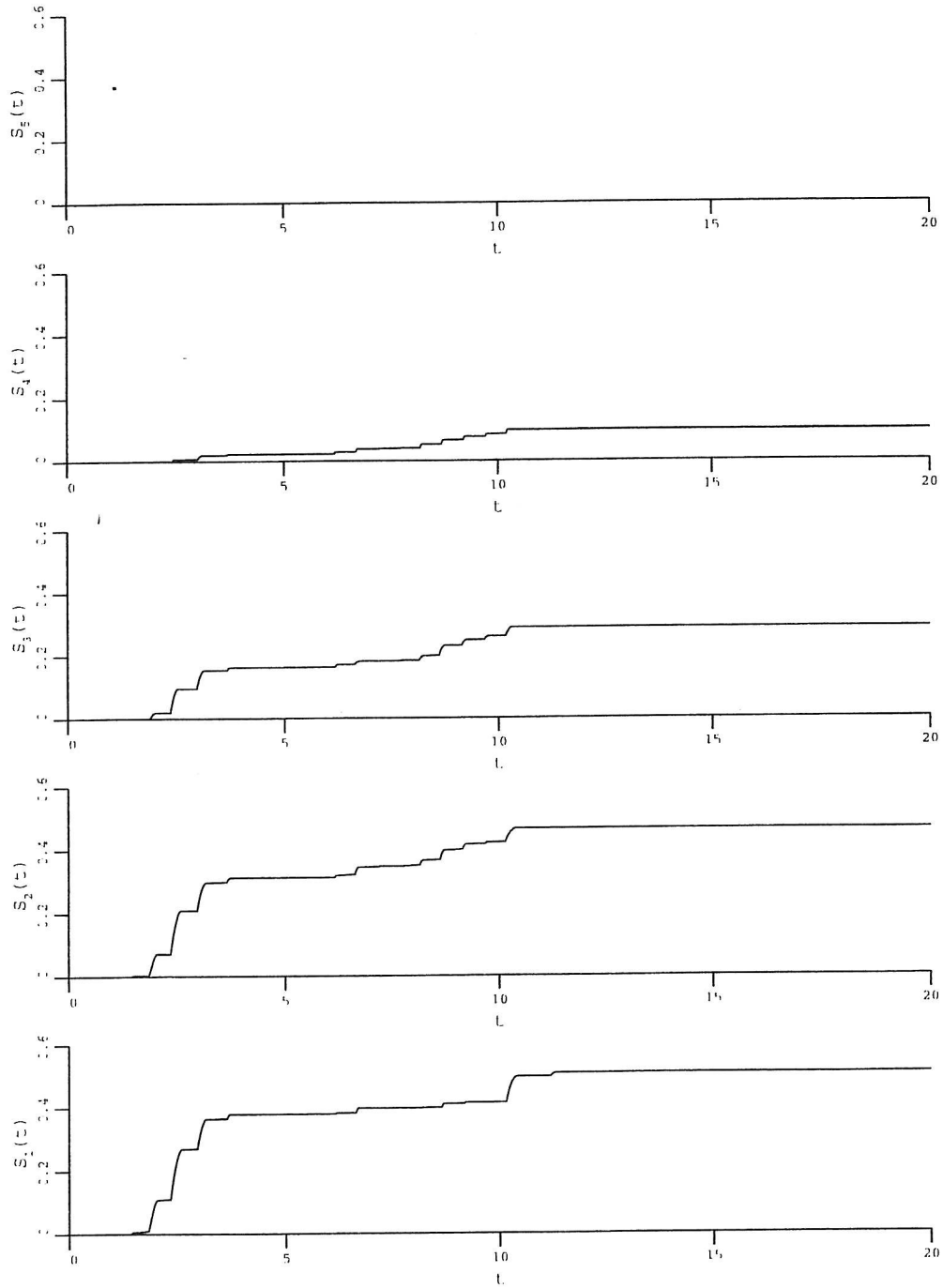


Figure 6. Instantaneous local softening values  $S_i(t)$  for the case described in Figure 5.



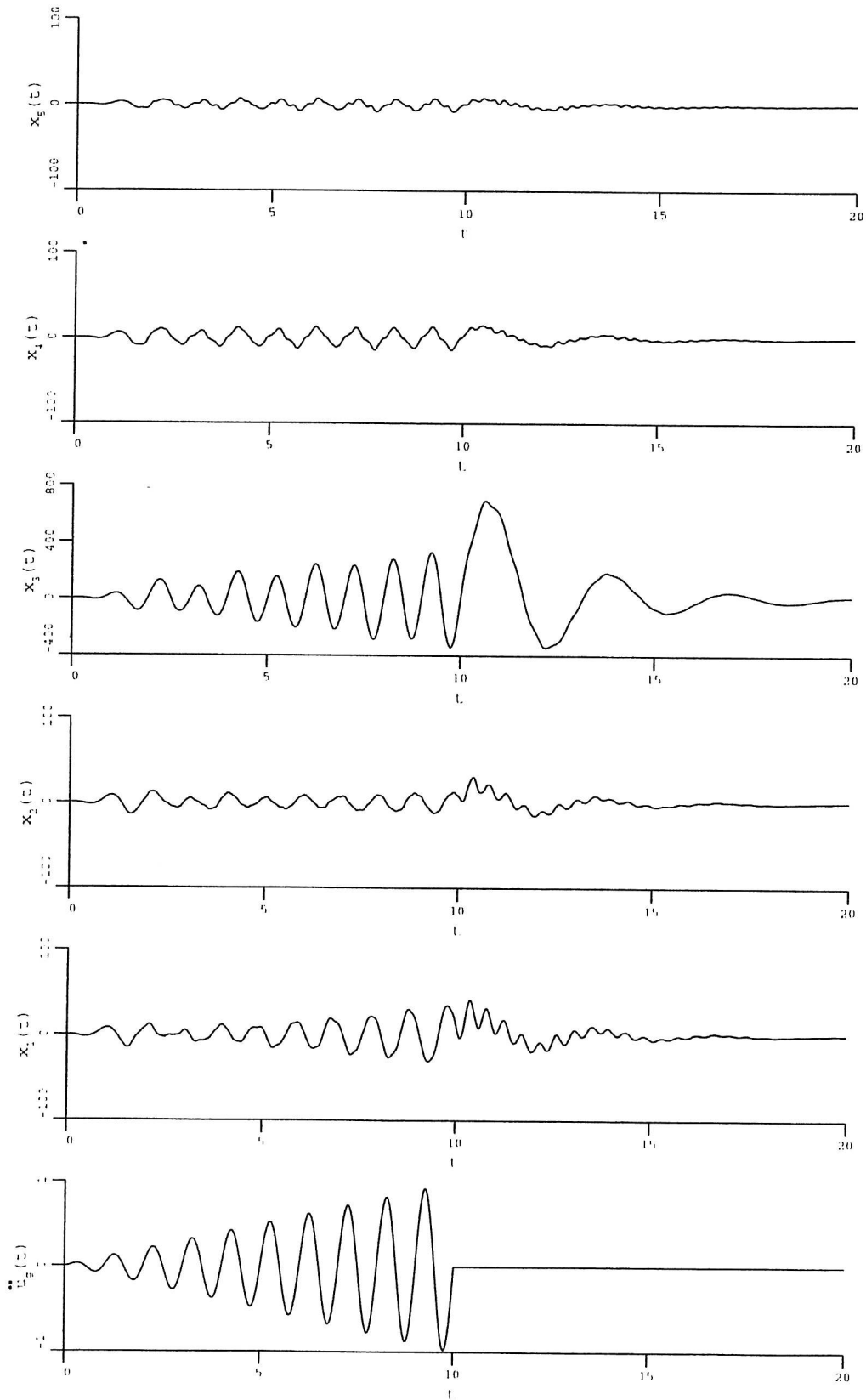


Figure 7. The sine ground acceleration (normalized to  $g$ ) and the response of the second structure in terms of relative displacements  $x_i(t)$  in mm.

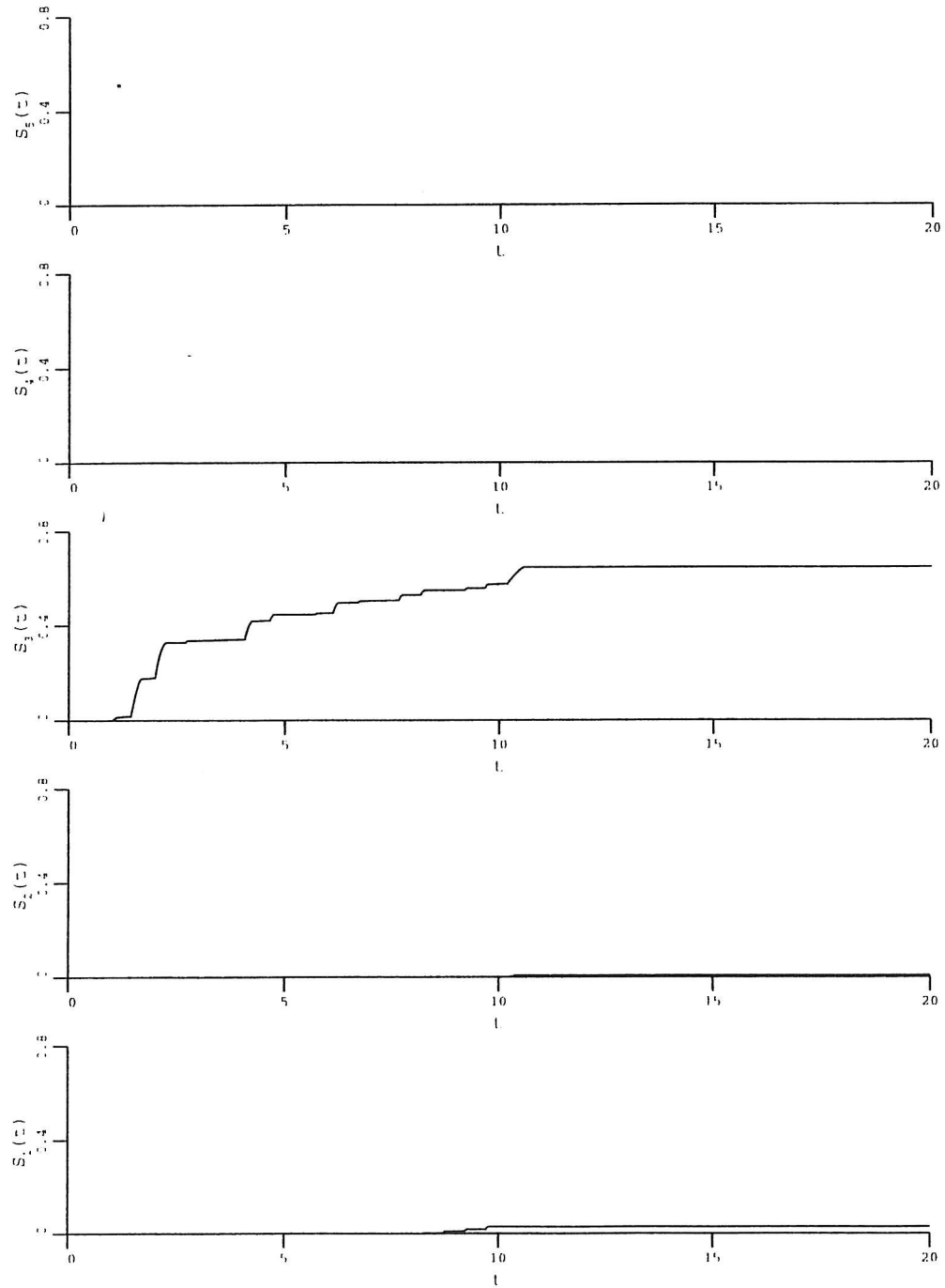


Figure 8. Instantaneous local softening values  $S_i(t)$  for the case described in Figure 7.

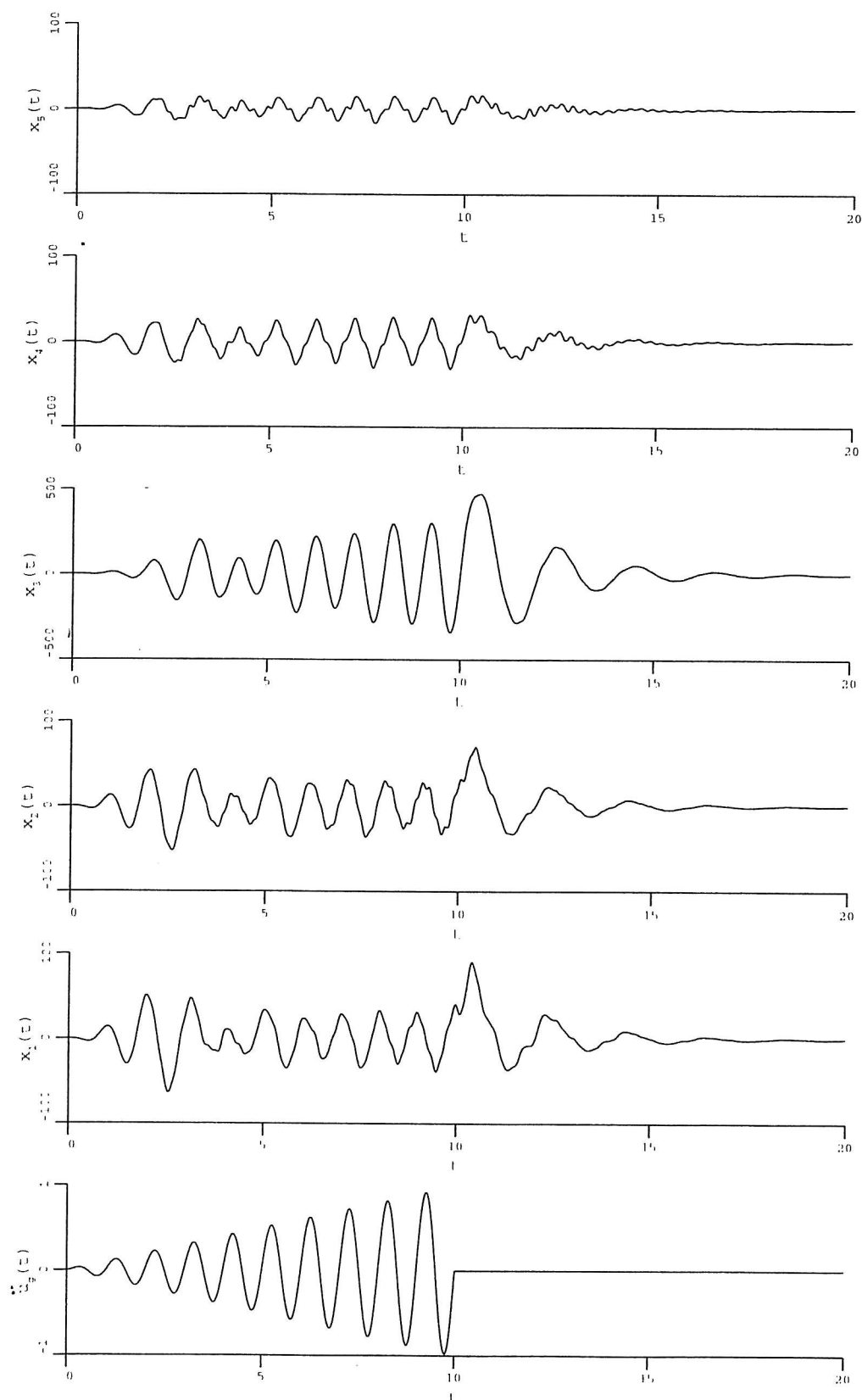


Figure 9. The sine ground acceleration (normalized to  $g$ ) and the response of the third structure in terms of relative displacements  $x_i(t)$  in mm.



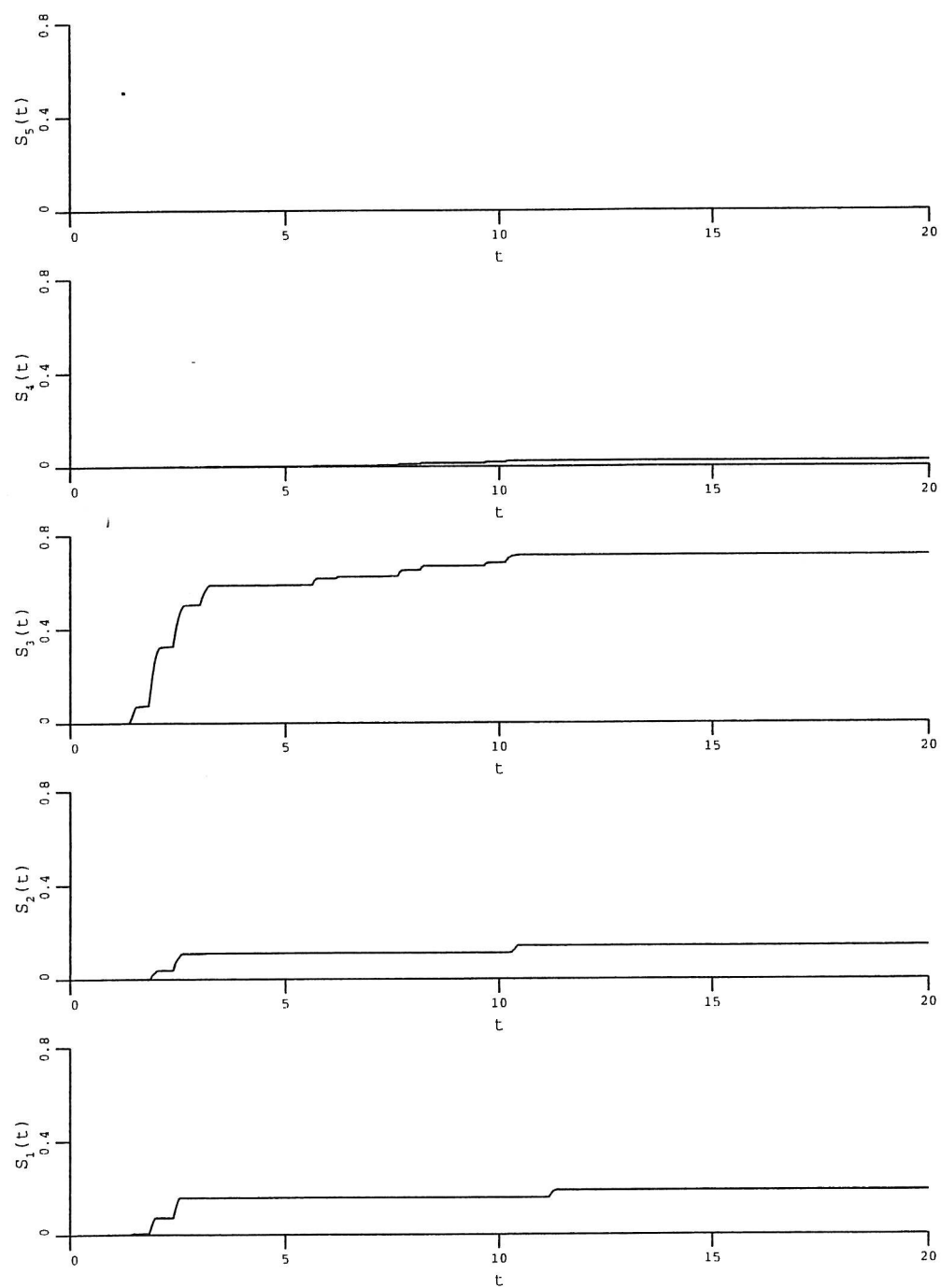


Figure 10. Instantaneous local softening values  $S_i(t)$  for the case described in Figure 9.

$$\ddot{u}_g(t) = E(t)V(t) \quad (14)$$

$$E(t) = \begin{cases} c_1 t & , \quad t < t_0 \\ c_2 e^{-c_3(t-t_0)} & , \quad t > t_0 \end{cases} \quad (15)$$

$$S_{VV}(\omega) = \frac{\omega_g^4 + 4\zeta_g^2 \omega_g^2 \omega^2}{(\omega^2 - \omega_g^2)^2 + 4\zeta_g^2 \omega_g^2 \omega^2} S_0 \quad (16)$$

For all the earthquakes applied,  $t_0 = 7$  sec,  $\zeta_g = 0.3$ ,  $S_0 = 1$  m<sup>2</sup> sec and the other envelope and spectrum parameters  $\omega_g$ ,  $c_1$ ,  $c_2$ ,  $c_3$  are given below for different types of earthquake simulations. It should be noted that a match in  $\omega_g$  and the  $j$ th frequency of the structure denotes an earthquake exciting the  $j$ th mode the most. Two different types of ground motions with statistically equivalent energy contents are utilized for excitation purposes. These are named as Type A and B. The ground motion parameters for Type A are  $\omega_g = 6.6$  sec<sup>-1</sup>,  $c_1 = 0.005$  sec<sup>-1</sup>,  $c_2 = 0.035$ ,  $c_3 = 0.2$  sec<sup>-1</sup>. The parameters for Type B are  $\omega_g = 19.2$  sec<sup>-1</sup>,  $c_1 = 0.00292$  sec<sup>-1</sup>,  $c_2 = 0.02044$ ,  $c_3 = 0.2$  sec<sup>-1</sup>. In these earthquakes the Kanai-Tajimi spectrum hits to its maximum value at the first and second frequency of the structure respectively, thus these modes are excited the most. The simulation of the stationary Gaussian stochastic processes is performed using the simulation procedure of Shinozuka and Deodatis (1991) which is based on spectral representation.

The results for one realization of the Type A and B earthquake excitations are given below. The peak accelerations in Type A and B earthquake simulations are 0.392g and 0.681g. These are strong earthquakes which may cause moderate to severe damage in RC structures.

The Type A earthquake excitation and the response of the three structures in terms of relative displacements are shown in Figures 11, 13 and 15. Figures 12, 14 and 16 are the corresponding local softening values and Table 3 lists the local MSDI under Type A and B excitations. All of these six figures and Table 3 are consistent with the properties of the structures, excitations and mode shapes. The weak structures at the third storey level recieved the maximum damage at this level. Since the Type A earthquake simulation excites the first mode the most, it causes the maximum damage compared to the Type B excitation, see Table 3 rows 1 and 4, and the lower part of the structure experiences the most damage for structure 1. Since the Type B earthquake model also excites the first mode, in addition to the second mode, all modes contribute to the distribution of damage in the structure. Localization of damage in case of a weak storey is once again illustrated in Figures 11-16 and Table 3 rows 2, 3, 5 and 6.

Table 3. Local MSDI due to the earthquake simulations

Excitation	Structure	$S_{M,1}$	$S_{M,2}$	$S_{M,3}$	$S_{M,4}$	$S_{M,5}$
$\sin(6.28t)$	Structure 1	0.420	0.303	0.136	0.008	0.000
	Structure 2	0.049	0.004	0.490	0.000	0.000
	Structure 3	0.202	0.141	0.599	0.000	0.000
$\sin(18.21t)$	Structure 1	0.048	0.008	0.026	0.023	0.010
	Structure 2	0.216	0.007	0.349	0.000	0.000
	Structure 3	0.048	0.003	0.267	0.014	0.000

Next, the sensitivity of the local MSDI to the local weaknesses is studied. First, the third storey stiffness is reduced gradually. The parameter  $r$  is defined to show the relative weakness in stiffness with respect to other storeys.

$$r = \frac{\omega_{0,3}}{(7\pi)\text{sec}^{-1}} \quad (17)$$

The variation in the local damage at the weak storey is followed, and the local MSDI  $S_{M,1}$  and  $S_{M,3}$  versus  $r$  are plotted in Figure 17. The excitation considered is the earthquake excitation of Type A. As shown in Figure 17, the local MSDI values for the third storey decreases as the third storey is assumed to be more and more strong in stiffness. On the other hand, the structure expects maximum damage at the first storey level if the design becomes more and more uniform, e.g.  $r \rightarrow 1$ .

A similar analysis is done for the strength sensitivity, the parameter  $q$  is defined to show the relative weakness in strength with respect to other storeys.

$$q = \frac{z_{0,3}}{24\text{mm}} \quad (18)$$

The local MSDI  $S_{M,1}$  and  $S_{M,3}$  versus  $q$  are plotted in Figure 18. The excitation considered is the earthquake excitation of Type A. A similar trend is observed in Figure 18 as in Figure 17. As the third storey has less strength, the damage is localized at the weak area, and, as all the storeys have the same strength, the damage concentrates on the first storey level.

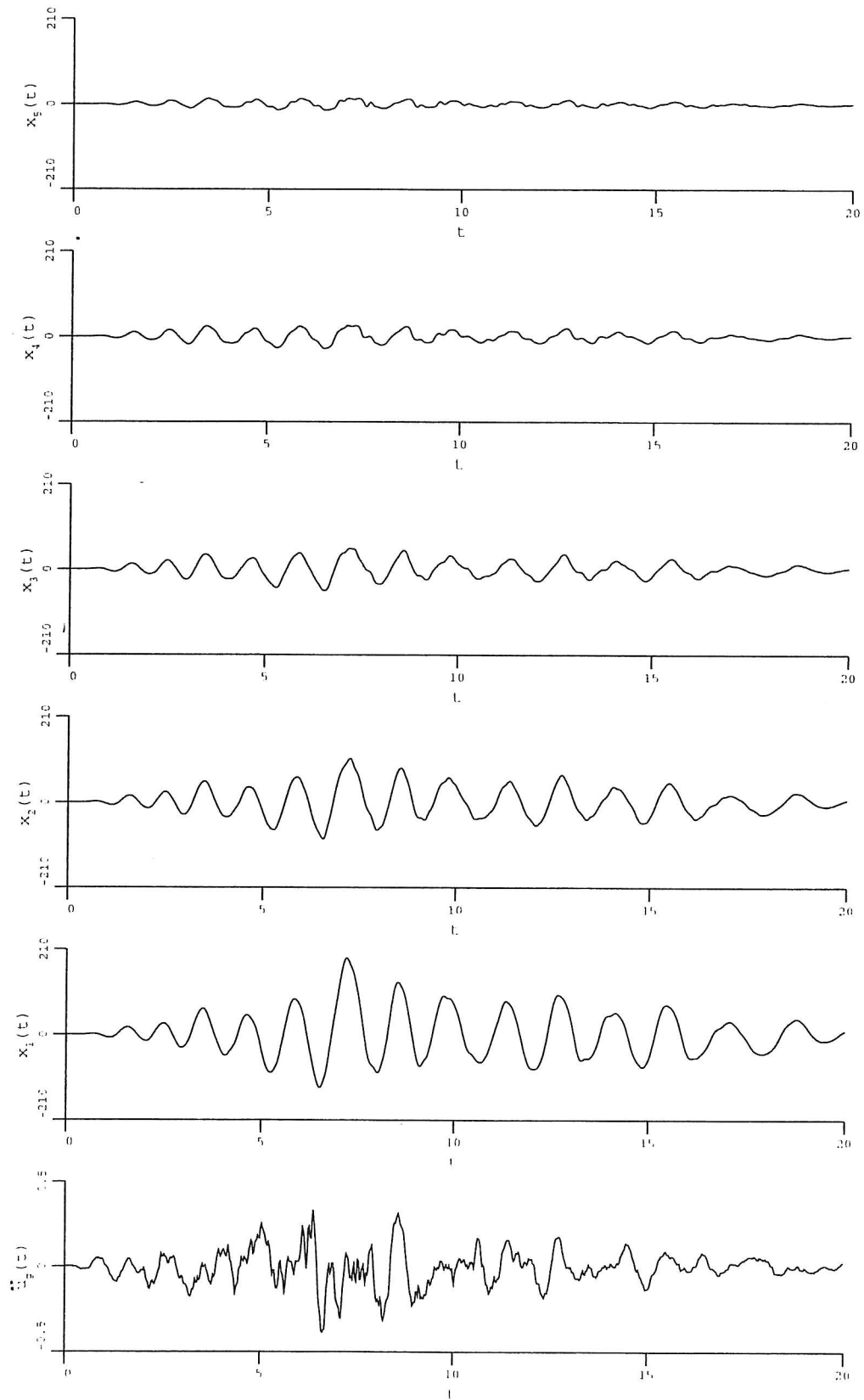


Figure 11. The earthquake ground acceleration (normalized to  $g$ ) of Type A exciting the first mode and the response of the first structure in terms of relative displacements  $x_i(t)$  in mm.



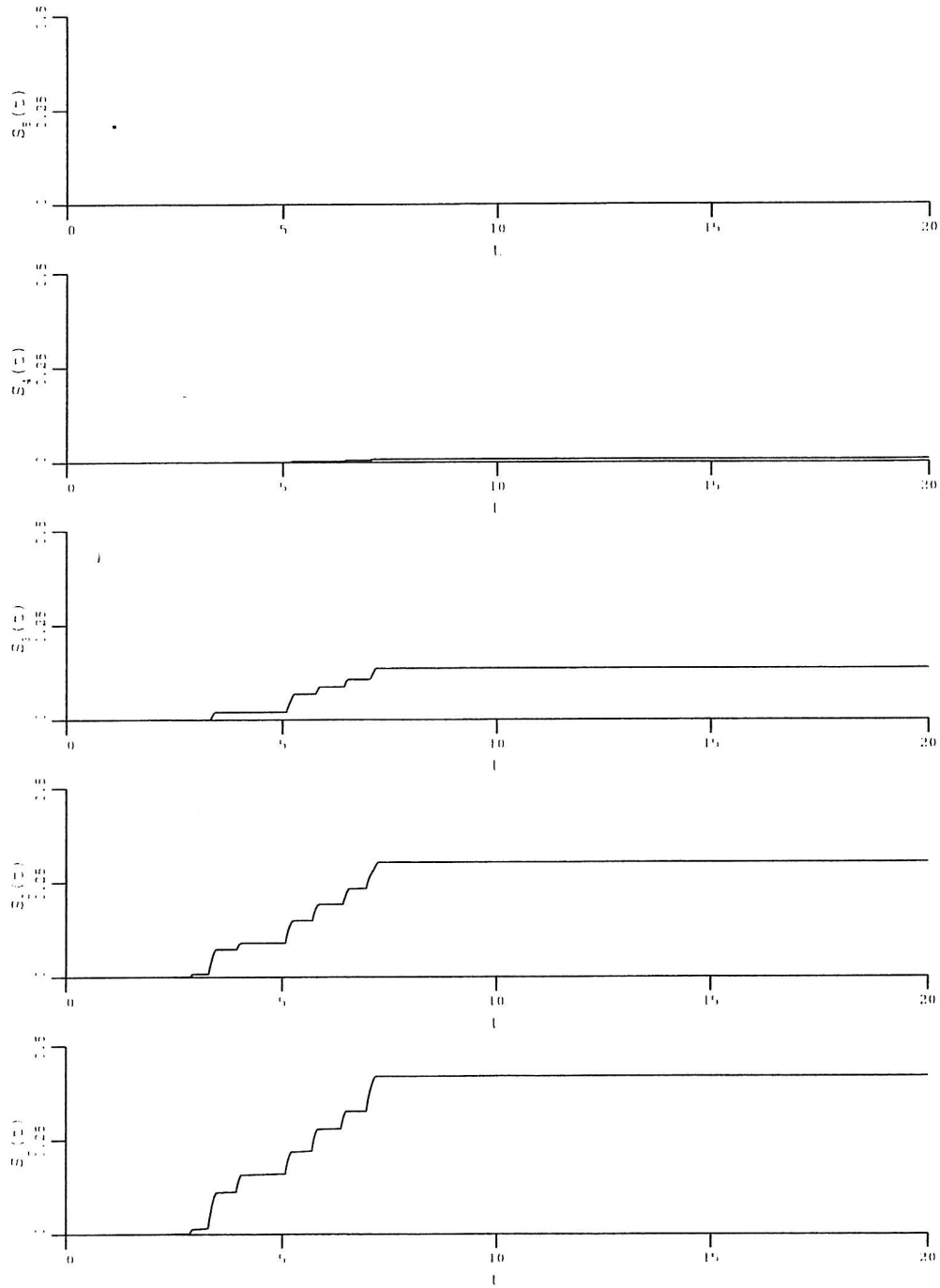


Figure 12. Instantaneous local softening values  $S_i(t)$  corresponding to Figure 11.

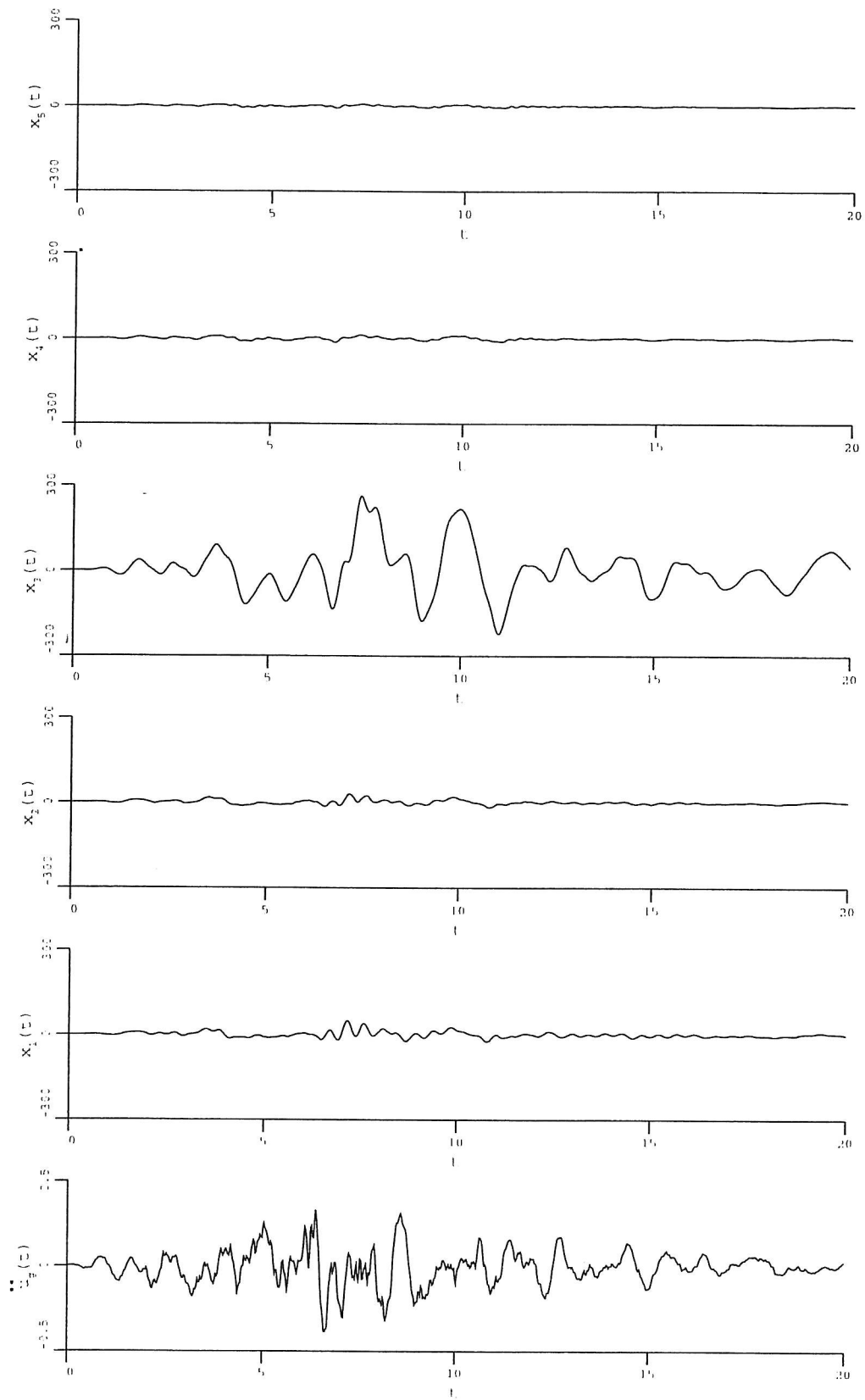


Figure 13. Type A earthquake ground acceleration (normalized to  $g$ ) and the response of the second structure in terms of relative displacements  $x_i(t)$  in mm.

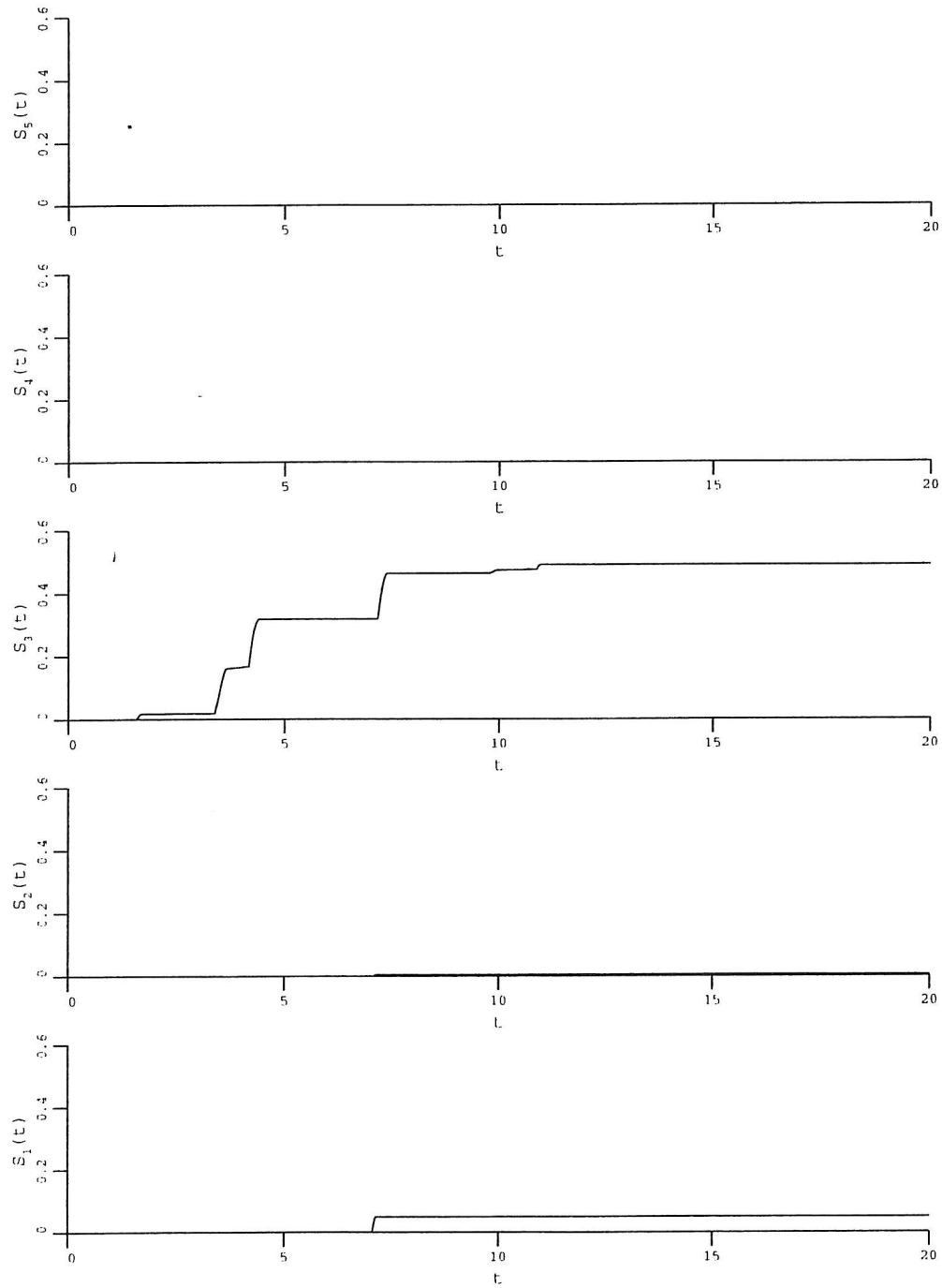


Figure 14. Instantaneous local softening values  $S_i(t)$  corresponding to Figure 13.

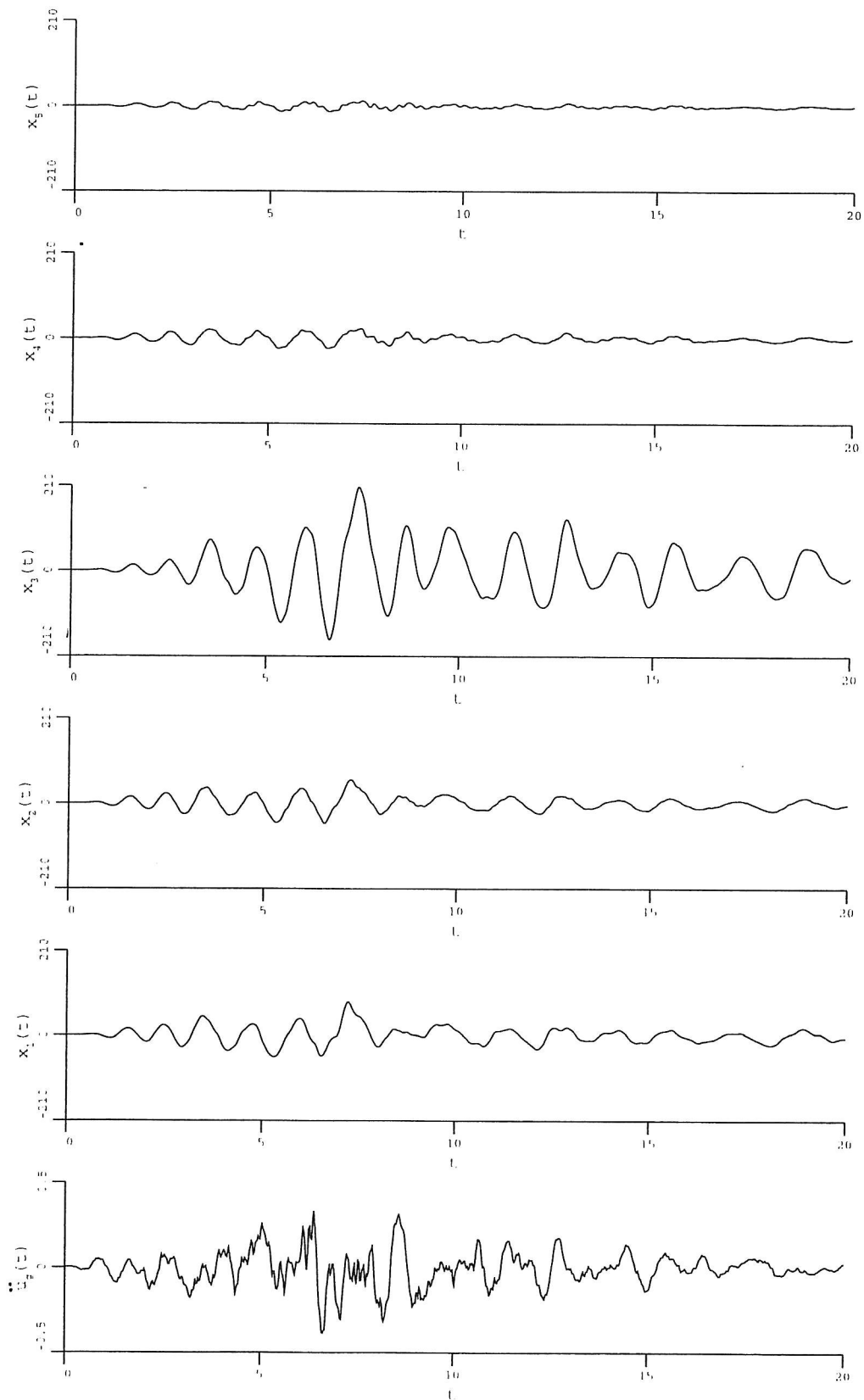


Figure 15. Type A earthquake ground acceleration (normalized to  $g$ ) and the response of the third structure in terms of relative displacements  $x_i(t)$  in mm.



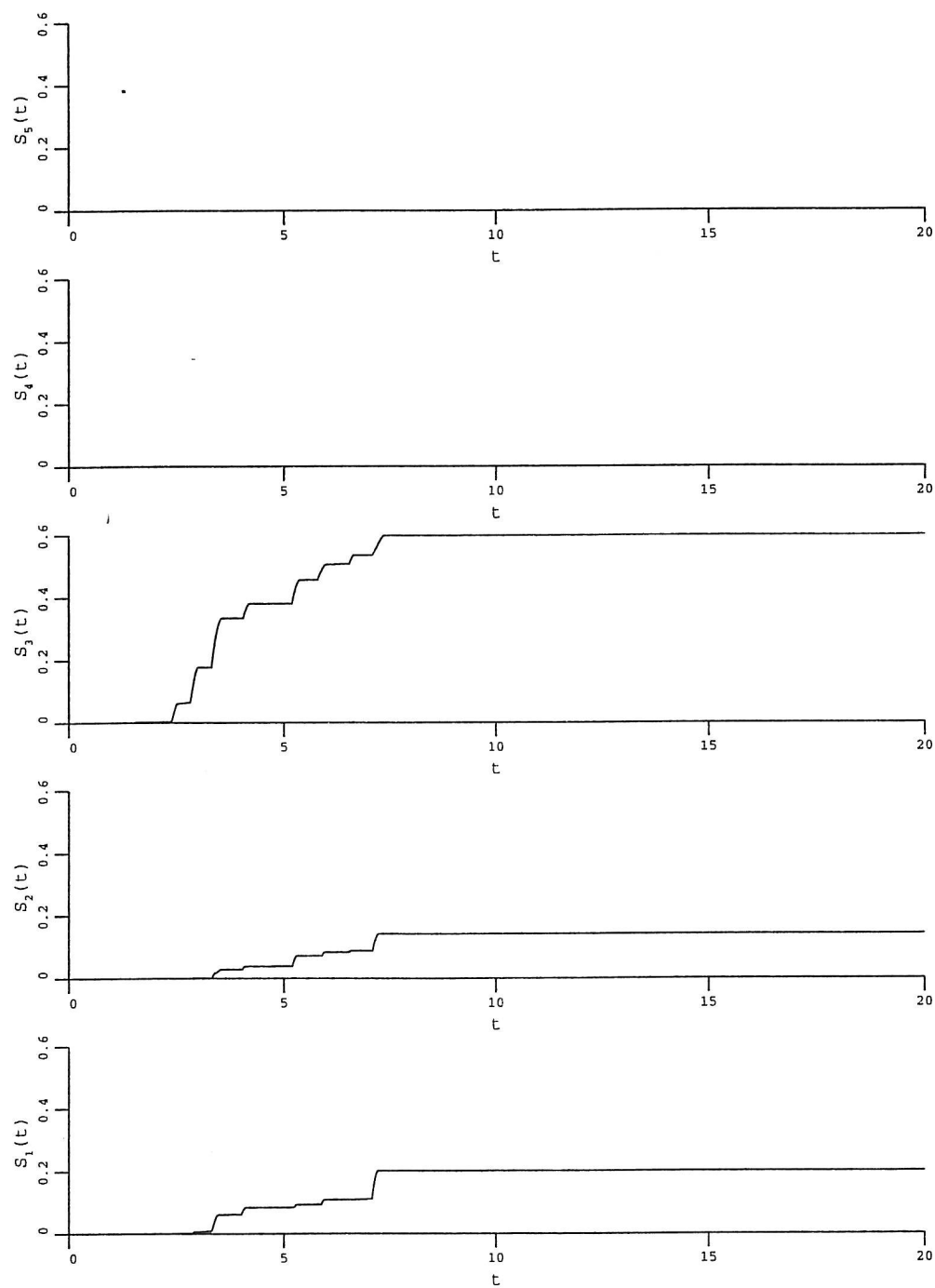


Figure 16. Instantaneous local softening values  $S_i(t)$  corresponding to Figure 15.

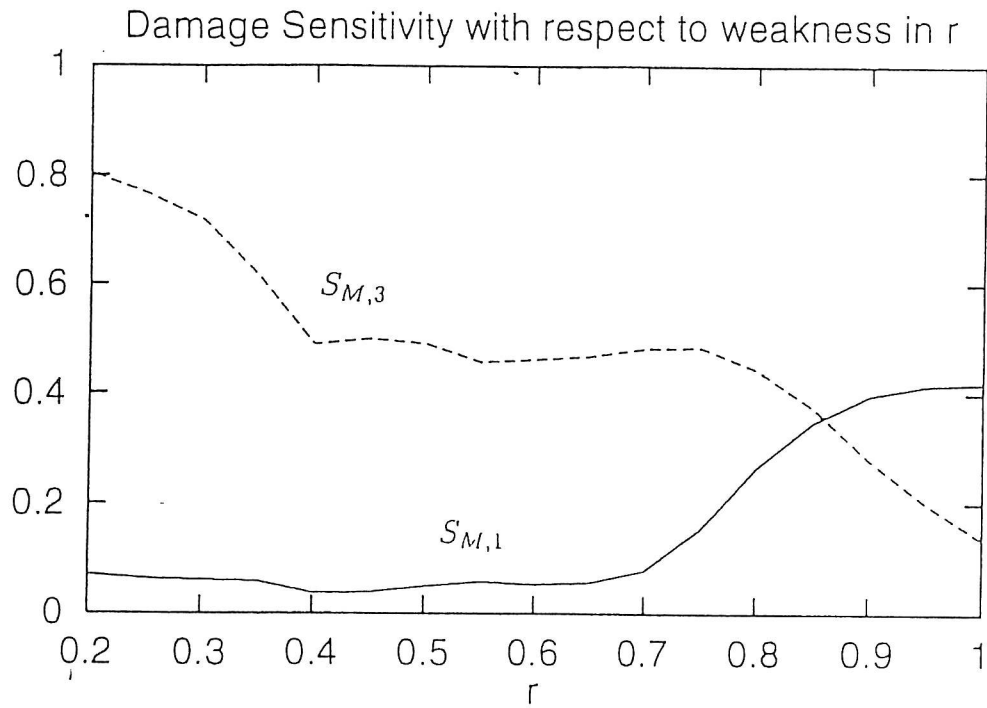


Figure 17.  $S_{M,1}$  and  $S_{M,3}$  as a function of  $r$  when the three structures are subject to earthquake excitation of Type A.

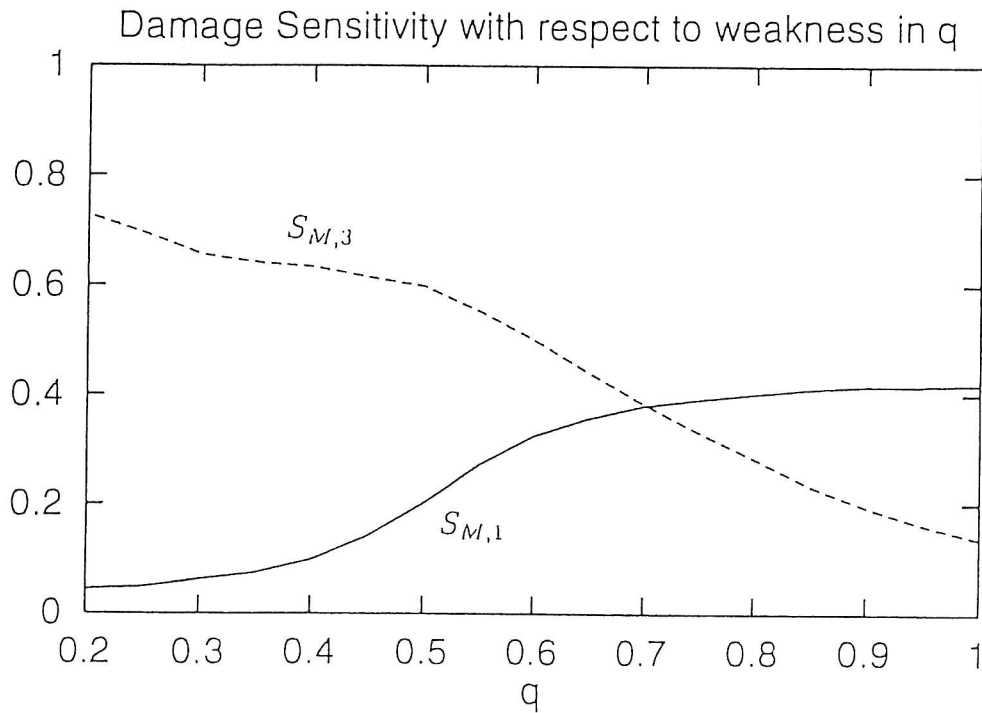


Figure 18.  $S_{M,1}$  and  $S_{M,3}$  as a function of  $q$  when the three structures are subject to earthquake excitation of Type A.

## 5. CONCLUSIONS

A hysteretic mechanical formulation to quantify local damage in reinforced concrete shear frames subject to earthquakes is derived. The performance of the model to explain the reasons of having midbroken structures due to horizontal ground motion is illustrated using three sample 5-storey shear frames subject to sinusoidal and simulated earthquake excitations. The three structures considered have the same structural and material properties except that the second has a local weakness in stiffness and the third has a local weakness in strength. From the numerical studies, it is observed that the collapses at the intermediate storeys can be attributed to these local weaknesses. The proposed model explains the distribution of damage by quantifying the damage using maximum softening damage indicators.

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Department of Building Technology and Structural Engineering  
Aalborg University, Sohngaardsholmsvej 57, DK 9000 Aalborg  
Telephone: +45 98 15 85 22    Telefax: +45 98 14 82 43